

Math Challenge #15

First Name: _____	Last Name: _____	Grade: _____
Teacher: _____	Parent's email: _____	

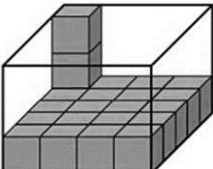
Squares and Cubes

Welcome to Math Challenge #15. This is the last challenge of this school year. Let's have fun with squares and cubes!

Kinder & First Grade: solve at least 3 problems.
Second & Third Grade: solve at least 7 problems.
Fourth Grade and above: solve at least 12 problems.

		<i>Answer</i>
1.	<p>Tiara uses cubes to make an X. How many cubes does she use to make two Xs?</p> <p style="color: blue;">She uses 9 cubes to make one X, therefore, she uses 9+9 or 18 cubes to make two Xs.</p>	<i>18 [cubes]</i>
2.	<p>Tim is building a structure with cubes. He has 5 cubes already. How many more cubes does he need to build the structure shown in figure 1?</p> <p style="color: blue;">Figure 1 has $2+3+3+3+2 = 13$ cubes. Since Tim has 5 cubes already, he needs $13 - 5 = 8$ more cubes.</p> <p style="font-size: small;">Figure 1</p>	<i>8 [cubes]</i>
3.	<p>a. What is the 11th shape in this pattern?</p> <p style="color: blue;">The pattern repeats every 5 shapes, so the 11th shape is square.</p> <p>a. What is the 15th shape in this pattern?</p> <p style="color: blue;">The pattern repeats every 6 shapes, so $15 \div 6 = 2 R3$, the third shape from the beginning is the same as the 15th shape or cube.</p>	<p><i>a. Square</i></p> <p><i>b. Cube</i></p>
4.	<p>Three identical cubes cost 10 cents more than one cube. How many cents does one cube cost?</p> <p style="color: blue;">If you draw them, you will notice that the 2 cubes cost 10 cents. Therefore, one cube costs 5 cents.</p>	<i>5 [cents]</i>
5.	<p>Laura is walking from M to S along the lines, picking up the letters in MATHEMATICS in order.</p> <p>The letters are scattered in the square grids. Each square is 1 yard by 1 yard.</p> <p>How long, in yards, is the shortest walk that Laura can take?</p> <p style="color: blue;">$7+3+4+1+7+2+3+3+2+1 = 33$</p>	<i>33 [yards]</i>

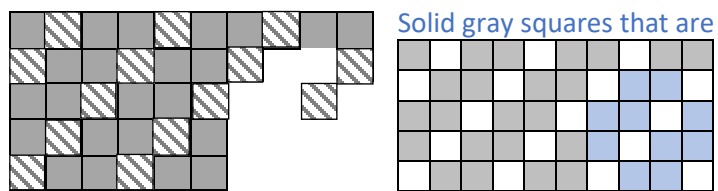
6. How many more cubes will be needed to fill the rectangular prism below?



The first layer has $4+4+4+4+4 = 20$.
 The second layer is missing $20 - 1 = 19$ cubes.
 The third layer is missing $20 - 1 = 19$ cubes.
 $19 + 19 = 38$

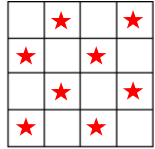
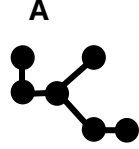
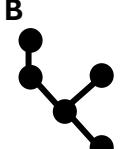
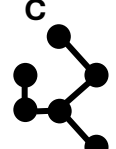
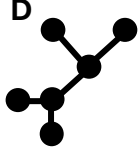
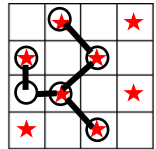
38 [cubes]

7. Two kinds of square tiles are used for a wall design with repeating patterns. The wall, when completed, will be 10 tiles by 5 tiles. How many more solid gray tiles are needed to complete the design?



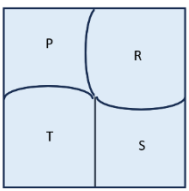
You can draw the model on the graph paper and count.
 Solid gray squares that are needed to finish the pattern 10.

10 [gray tiles]

8.  **A**  **B**  **C**  **D**  

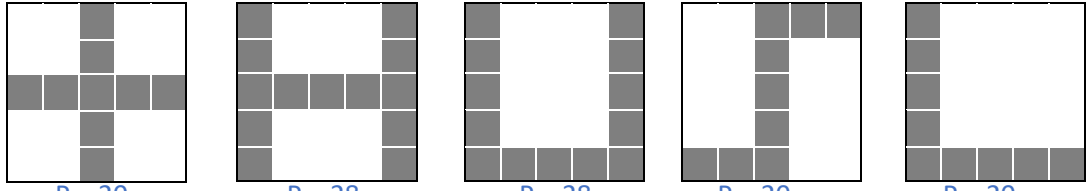
The square has an alternating pattern with the stars. You can use transparent paper/Ziplock to sketch the shapes and try it out.
 Or
 Shape A covers at most 4 stars.
 Shape B covers at most 4 stars.
 Shape C covers at most 5 stars.
 Shape D covers at most 4 stars.
Only C covers 5 stars.

1

9. 

You can use transparent paper to try it out.
 You can also analyze the edges of the shapes. 4 pieces must create a square, which means there should be right angle with two straight edge meeting at it in each shape.
 Additionally, Shape P has 2 caved in edges; Shape Q 1 caved in, 1 bulge out; Shape R 2 bulge out edges; S 1 straight, 1 caved in; T 1 straight, 1 bulge out. S and T must be paired together, as they are the only ones having additional straight edge. We must use both S and T to build our square. Thus, we now have 1 caved in edge, and 1 bulge out edge to fill. Try and error, you can find that the only option to build a square is using P, R with T and S.
Piece Q will not be used.

Q

10. 

$P = 20$ $P = 28$ $P = 28$ $P = 20$ $P = 20$

The perimeter of the paper is $5 \times 4 = 20$ units.

3 [shapes]

11. You can take a square origami paper to try it out. When you fold B over EF, C over FG, D over HG, A over EB, you will get a smaller square EFGH. All the point B, C, D, and A will meet exactly in the center, so the area of the square EFGH is half of the square ABCD. Area of ABCD is 16 square units, thus, the smaller square EFGH has an area of $\frac{1}{2} \times 16 = 8$ square units.

12. The area of the shaded region and the added rectangle is equal to $(\frac{1}{2} \times 10 \times 7.5) = 37.5$. The area of the shaded region is $37.5 - \frac{1}{2}$ of the area of one square. $37.5 - \frac{1}{2} \times 5 \times 5 = 37.5 - 12.5 = 25 \text{ in}^2$. Or you can rotate the triangle from the bottom right square and attach it to the right trapezoid in the top layer, it will be a completely shaded square, so the area is $5 \times 5 = 25 \text{ in}^2$.

13. The perimeter of the shaded region is the same as the perimeter of the bigger square. The side of the bigger square is 7 cm, so the perimeter is 28 cm.

$2 + 7 + 7 + 2 + 5 + 5 = 28$

14. You can think how to number the faces on a cube net. Opposite to 4 is 1.

15. The pentagon must have 7 unit². Since each triangle is 1 unit², the square must be 8 unit².

16. The surface areas of the two solids are the same. Hence the same amount of paint is required to cover them. Therefore, it would take **7.5 liters** of paint to cover the surface of the second solid.

17. Since there are 256 small squares, the original square was made of 16 by 16 small squares. The side of the original square must be: $16 \times 1.5 \text{ inches} = 24 \text{ inches}$. Thus, the perimeter is $24 \times 4 = 96 \text{ inches}$.

18. a. There are 3 faces for each cutout and there are 8 cutouts: 24 faces. Original faces: 6. Thus, $24 + 6 = 30$ faces.
b. The surface area after the cuts is the same as the original surface area, which is $3 \times 3 \times 6 = 54$ sq cm.